# Diversi cation in Multi-Product Choices: Bias or Rational Utility Maximization?

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considerable variation from one consumption occasion to the next. This element of randomness was dramatized in an iconic ad campaign for Almond Joy and Mounds candy bars. As the jingle suggests, ccnsumers are aware that their future preferences are uncertain...so they are better o having a choice.

When shopping for products for future consumption, it is rational for consumers to consider this preference uncertainty. For example, a consumer shopping for soup might buy multiple cans of vegetable soup (their favorite) but also include other alternatives that they prefer on occasion; for example, chicken soup for when they are not feeling well or tomato soup to pair with a grilled cheese sandwich. Consumers therefore often buy multiple products (in di erent quantities) in a category, which they store at home to be consumed later.

The primary nding in the literature addressing the selection of multiple products in a category is that consumers consistently include \too much" variety when choosing a set for future consumption (Simonson, 1990; Read and Loewenstein, 1995). This nding is known as *diversi cation bias* and is based on the empirical regularity that consumers include more variety (i.e., more di erent product alternatives) when choosing a set of products for future consumption | known as *simultaneous choice* | compared to products chosen one-at-a-time on each consumption occasion | known as *sequential choice*. Strangely, the experimental evidence of diversi cation bias imposed an extraneous requirement on simultaneous choice. Participants were required to precommit to the exact order in which all products in their set would be consumed. In actuality, consumers can choose any can of soup they have at home...or none of them (the same is true of any product category). This unnatural precommitment requirement suppresses the impact of future preference uncertainty and reduces simultaneous choice to a forecasting exercise. Further, Reed and Lowenstein found that 44% of their simultaneous choice participants wanted to change the precommitted order during the consumption sequence, evidence that the requirement prevented them from accommodating their true preferences once uncertainty is resolved, as they would naturally. Because imposing such an arbitrary, utility-reducing requirement is inconsistent with actual behavior, neither our utility-maximizing models nor our empirical tests impose any restrictions on consumption sequences. To make this distinction clear, we refer to a set selected without any consumption precommitment restrictions as *multi-product-choice.*

Diversi cation bias refers to the product variety in simultaneous choice compared to the variety across an equal number of sequential choices. Each sequential choice involves the selection of a product | any product | from the full assortment, so it is not restricted by the simultaneous choice set nor previous consumption choices from that set.

In contrast, multi-product choice involves the construction of a set of products from which subsequent consumption choices can be made. This set is then reduced by one unit after each consumption choice, leaving fewer products for subsequent consumption choices. Because previous consumption choices impact subsequent choices, forward-looking dynamic ]TJ 0 -13.55 Tro f8(h)-33(consumao72)

are observed immediately before consumption). A myopic consumer would select alternative 1 if $U_1 + \epsilon_{11} > U_2 + \epsilon_{21}$ and select alternative 2 if $U_2 + \epsilon_{21} > U_1 + \epsilon_{11}$ (ties can be broken arbitrarily). However, a forward-looking consumer would consider both the current utility and the *expected future utility*. Letting $V(q)$ represent the value of expected total future utility for a vector of quantities $q$, the strategic consumer chooses the alternative that maximizes $U_1 + \epsilon_{11} + V(1;1;0;0)$ vs. $U_2 + \epsilon_{21} + V(2;0;0;0)$. Note that the future values are different because they incorporate different reductions in future inventory. The same "current utility" plus "expected future utility" comparison is done at each subsequent consumption occasion. In general, the hard part is determining a manageable expression for the expected future utility, or "value," function $V$.

This framework necessarily abstracts shopping and consumption behavior. For example, the total number of products, $n$, is assumed to be exogenous.[2] Clearly, factors such as trip type (major versus fill in, cf. Kollat and Willet, 1967) and incentives for multiple purchases could affect $n$. Also, the consumer's deterministic component of utility could be a function of store-specific factors such as price, or time-varying factors such as satiation. Introducing these complexities would not only greatly complicate the modeling effort, but would also obscure the basic insights on which our propositions are based.

If the random errors are assumed to follow a standard (zero-mean) Gumbel distribution — as one model in §3.2 will show that the weighting of these objectives may not be equal. The consumer's utility-maximizing set must balance these two objectives.

**Proposition 1.** *Consumers' multi-product choices (i.e., set selection) will reflect a tradeoff between the intrinsic utility of products in the set and the consumption flexibility afforded by the structure of that set.*

In order for (3.1) to capture the expected future value of a given set, that set would have to be consumed according to a policy that maximizes its value. Suppose that, on the $t^{th}$ consumption occasion, the set has $q_{it}$ units of alternative $i$ remaining. Then the policy that maximizes the set's future value is to consume the alternative that maximizes $ln(q_{it}) + \varepsilon_{it}$ (see Appendix A). Observe that this consumption policy incorporates each product's current inventory, ($q_{it}$) and its random component, $\varepsilon_{it}$, but not its deterministic component, $U_i$

[redacted]

satisfying $x_i \leq k_i$ and (ii) the number of times that the outside option is consumed, $x_0$, which must satisfy $x_0 = T - \sum_{i=1}^{M} x_i$. Then

$$V_T(k_1, k_2, \ldots, k_M) = \ln \left[ \sum_{(x_0, x_1, \ldots, x_M) \in S_T(k_1, k_2, \ldots, k_M)} \frac{T!}{x_0! x_1! \cdots x_M!} e^{\sum_{j=0}^{M} x_j U_j} \right.$$

Those participants were presented with an assortment of 12 products from their preferred snack category, assortments of products stocked in local vending machines and pretested in a pilot study. The same product assortment was offered to every participant who selected \candy" or \salty snack," respectively. Presentation order was randomized.

We elicited participants' long-run choice probabilities for products in the assortment by asking them to \[*p*]lease assign a choice percentage to each product, so that they add up to 100 (it's OK to assign a choice percentage of 0 to a product)." These long-run choice probabilities were subsequently used to compute participants' utilities for products in the assortment. Next, participants were then asked to identify:

[i] their favorite product in the assortment: \Of the candies available in the vending machine, which would you say is your favorite?"

[ii] their second favorite product: \Of the candies available in the vending machine, which would you say is your second favorite? (it's OK to pick a product that you might not actually choose during the year)"

[iii] their third favorite product: \Of the candies available in the vending machine, which would you say is your third favorite? (again, it's OK to pick a product that you might not actually choose during the year)"[5]

Based on pretesting, we determined that eliciting more than three ordered favorites was cognitively taxing and yielded unreliable data. These ordinal preferences were used to construct participant-specific multi-product choice sets in a way that makes the empirical analysis tractable.

We then asked a series of questions about category usage rate, attitudes, and perceptions. Neither these responses nor the demographic responses collected previously were analyzed in this paper; however, they are available as additional explanatory variables if required. Next, participants made their multi-product choices. The choice task was substantially different for 3-, 4-, and 5-product sets. Using participant's self-reported favorites, multi-product choices were constrained by requiring that $k_f \geq k_{f+1}$, where $k$ represents the quantity in the choice set and the subscript indicates favorite ordering. Applying this constraint results in three possible 3-product choice sets, five possible 4-product choice sets, and seven possible 5-product choice sets. Table 1 shows the possible choice sets, including choice set notation in parentheses as well as the exact language from the questionnaire. Observe that, for 4- and 5-product choice sets, participants could include a product alternative that was not identified as one of their three favorites. Observe also that, for 4- and 5-product choice sets, different sets may include the same *variety*; defined here as the number of different alternatives available in the set. For example, (3,1,0,0) and (2,2,0,0) both have two product alternatives and therefore the same amount of variety.

## 4.2 Analysis of Multi-Product Choice Experiment

A sample of 5,140 qualifying completed questionnaires was collected.[6] Questionnaires were then screened to ensure that the product alterative to which the participant assigned the highest long-run choice probability was included among their three favorites. This screen was designed to

[5] Note that participants were prevented from duplicating favorite selections.

[6] Questionnaires were qualified if:

[i] the participant's preferred snack category was either \candy" or \salty snack."

[ii] the participant responded correctly to an attention check question within the questionnaire.

[iii] the participant was not a 'speeder;' i.e., did not complete the questionnaire in less than 1/3 of the median completion time.

Table 1: Multi-Product Choice Sets

| | |
|---|---|
| 3-Product Set | |
| of 3 candies/salty snacks below for the next 3 occasions you eat a single- | "Now, choose one of the sets serving candy/salty snack." |
| | 4-Product Set |
| ies/salty snacks below for the next 4 occasions you eat a single- | "Now, choose one of the sets of 4 cand serving candy/salty snack." |
| ou eat a single- | "Now, choose one of the sets of 5 candies/salty snacks below for the next 4 occasions yo |

ensure test/retest reliability of participants' long-run preferences. After applying this screen, we analyzed the remaining 4,191 questionnaires.[7]

Table 2: Choice Set Variety

| | 3-Product Set | | 4-Product Set | | 5-Product Set | |
|---|---|---|---|---|---|---|
| | Candy | Salty Snack | Candy | Salty Snack | Candy | Salty Snack |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Table 2 summarizes the actual variety of the multi-product sets, organized by set size and category. We find that, although maximum variety increases with the choice set size, actual mean variety increases much less, median variety increases for only one category set size combination, while modal variety does not increase at all. Diversification bias implies that variety will increase with choice set size (Simonson and Winer, 1992). The data do not support this.

Recall Proposition 1's tradeoff between the intrinsic utility of products in the choice set and the consumption flexibility afforded by the choice set's structure. Intrinsic utility is the sum of expected utilities of products in the choice set. We determined each participant's expected utilities $U_j = ln(p_j/p_1)$, where their favorites are ordered by the indicator variable $j$ and $p_j$ is the the long-run choice probability of their $j^{th}$ favorite. Consumption flexibility is captured by the expression $ln(n!) \quad \sum_{i=1}^{M} ln(k_i!)$ from (3.1).

To assess how well Proposition 1's tradeoff explains multi-product choices, we estimated four multinomial logit [*MNL*] choice models. *MNL* choice models were estimated separately for candy and salty snack categories' 3-, 4-, and 5-product choice sets.[8] For each category set size combination, we estimate models *A* through *D*:

[i] *A* is a two-parameter model with separate intrinsic utility and consumption flexibility coefficients to allow differential weighting

[ii] *B* is a one-parameter nested model in which the intrinsic utility and consumption flexibility coefficients are restricted to be equal

[iii] *C* is a one-parameter nested model in which the consumption flexibility coefficient is restricted to be zero.

[iv] *D* is a multi-parameter choice set intercepts model.[9]

[7] We checked the robustness of our results to this reliability screen. Specifically, we replicated our choice set analyses both after applying a relaxed screening criterion (eliminating only the few questionnaires for which all three favorites were assigned zero long-run choice probabilities) and after applying two more stringent screens. Regardless of the screening criteria applied, our results were substantially the same.

[8] Recall that the choice set configurations are different for 3-, 4-, and 5-product sets. This prevented us from pooling over set sizes. Further, we estimated separate models for candy and salty snacks to allow for systematic differences between categories.

[9] Regardless of set size, the single-variety set | either (3,0,0), (4,0,0,0), or (5,0,0,0,0) | is the baseline for choice set intercepts model estimation.

There are no clear alternatives in the literature to the tradeoff model $A$, so we estimate nested models and less parsimonious choice set intercepts models for comparison.[10] Model $B$'s parameter restriction reflects the $RMPC$ base case, in which a product is consumed on each consumption occasion. Comparing model $C$ to the tradeoff model $A$ permits us to assess the incremental predictive contribution of consumption flexibility. The choice set intercepts model $D$ incorporates both consumption flexibility and intrinsic utility via set structure, so we would expect it to offer comparable predictive accuracy to the more parsimonious tradeoff model $A$. Model fits are assessed in sample using $AIC$ and $BIC$. Model fits are assessed out of sample using a ten-fold validation. For this validation, we randomly partitioned each dataset into ten equally-sized subsets. Each subset then served as a validation sample, while the other nine were used for estimation. Hit rates were averaged over the ten validation samples.[11]

Table 3 shows fit statistics and parameter estimates for the $MNL$ choice models. Models $A$ through $D$ are arranged in vertical panels; categories and set sizes are arranged horizontally. Across the six category × set size combinations, model $A$ explains multi-product choice data better and offers superior predictive accuracy compared to the other three models. In fact, model $A$ dominates the two nested models, $B$ and $C$, and is clearly superior to the choice set intercepts model $D$.[12] Thus, the $MNL$ choice model fits provide support for Proposition 1 and evidence that consumers' multi-product choices are consistent with a tradeoff between intrinsic utility and consumption flexibility. Note that, across categories and choice set sizes, all model $A$ parameter estimates in the top panel of Table 3 are positive and significant at the 0.001 level. Comparing those parameter estimates, we find that the consumption flexibility parameter estimate is higher than the intrinsic utility parameter in all six category × set size combinations. Testing the difference between the two parameters (incorporating standard errors of the parameter estimates), we find that the difference is statistically significant for five of the six category × set size combinations. The higher weighting of consumption flexibility vis-a-vis intrinsic utility in multi-product choices is consistent with $RMPC$'s more general model, which allows for an outside option. The implication is that consumers do not eat candy or a salty snack on every consumption occasion, but rather consume those snacks less frequently.

We now consider the predictive contribution of consumption flexibility (resulting from the structure of the choice set) after accounting for intrinsic utility by comparing the hit rates of model $A$ and model $C$. As noted above, the hit rate of model $A$ is higher than model $C$ for every category × set size combination. Across the combinations, the mean increase in hit rate from adding consumption flexibility (i.e., model $A$ compared to model $C$) is 14.4% — a material improvement in the explanation of multi-product choice. To put that improvement in context, consider that the mean hit rate of model $C$ (without consumption flexibility) is only 25.0%. More importantly, the hit rate of model $C$ *above what would be expected by random chance* is only 2.4%.[13] Finally, we apply model $A$'s parameter estimates to the data for each category × set size combination to see if we recover the aggregate choice set variety patterns. Table 4 shows the choice set varieties predicted by model $A$ with the corresponding actual choice set varieties, as reported in Table 2, in parentheses.

Observe that model $A$'s predictions recover the aggregate patterns of Table 2's actual choice

[10] Note that the econometric models of multi-product choice are estimated using time series purchase data.

[11] In sample hit rates were also computed. They are very similar to hit rates computed in the ten-fold validation and result in the same conclusions.

[12] In five of the six category × set size combinations, model $A$ offers a higher hit rate and lower $AIC$ and $BIC$ than all other models. For 4-product candy choices, however, the 5-parameter choice set intercepts model $D$ has a lower hit rate (38.8%<39.3%) and slightly lower $AIC$ (1201.2<1202.1) compared to model $A$, but a higher $BIC$ (1217.2>1210.1).

[13] The hit rate that would be expected by chance is simply the inverse of the number of alternative sets shown in Table 1.

Table 3: Multi-Product Choice Set Model Estimates

| | Choice Set Size = 3 | | Choice Set Size = 4 | | Choice Set Size = 5 | |
|---|---|---|---|---|---|---|
| | **Candy** | **Salty Snacks** | **Candy** | **Salty Snacks** | **Candy** | **Salty Snacks** |
| obs = | 413 | 959 | 402 | 972 | 413 | 1032 |
| **Model A – Intrinsic Utility + Consumption Flexibility** | | | | | | |
| Hit Rate ‡ | 51.8% | 53.7% | 38.8% | 39.2% | 25.9% | 26.6% |
| Log Likelihood | -406.8 | -927.5 | -599.0 | -1448.0 | -769.6 | -1896.4 |
| AIC | 817.5 | 1859.0 | 1202.1 | 2900.0 | 1543.1 | 3796.9 |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Table 4: Predicted vs. Actual* Choice Set Variety

| | 3-Product Set | | 4-Product Set | | 5-Product Set | |
|---|---|---|---|---|---|---|
| | Candy | Salty Snack | Candy | Salty Snack | Candy | Salty Snack |
| 1 | 13.6% (22.8%) | 5.6% (19.3%) | 9.0% (15.9%) | 9.1% (15.3%) | 15.3% (16.1%) | 13.7% (14.0%) |
| 2 | 18.2% (28.6%) | 17.8% (28.1%) | 19.2% (27.0%) | 20.2% (28.4%) | 22.8% (23.4%) | 20.5% (23.0%) |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| [illegible] | | | | | | [illegible] |

[illegible]

set variety. Specifically, we find that, although maximum variety increases with the choice set size, mean predicted variety increases much less, and median and modal predicted variety of three product alternatives does not increase at all.

# 5 Longitudinal Experiments

Proposition 2 specifies a rational consumption policy for multi-product choice sets. This consumption policy — that available product alternatives will be chosen for consumption in proportion to their current inventory — is implied by *RMPC*. As products are consumed one-by-one, the effect of this policy is to maintain flexibility by probabilistically balancing the inventory of product alternatives in the set. To test Proposition 2, we conducted two longitudinal experiments [i] to determine whether actual consumption patterns are consistent with Proposition 2, [ii] to test the robustness of Proposition 2 to different choice set concentrations, and also [iii] to test variety seeking as an alternative explanation for actual consumption choices.

## 5.1 Experimental Design

The first experiment was conducted to determine whether participants' consumption choices are consistent with Proposition 2. The second experiment was conducted to test the robustness of Proposition 2 across a variety of inventory levels and to test for variety seeking in consumption choices. Following Simonson (1990), these experiments involved students consuming snack products once or twice per week over a series of consumption occasions. The experimental design was approved for human participants by our University's Institutional Review Board.

The first phase of these longitudinal experiments was exactly the same preference elicitation process used in the large-scale multi-product choice experiment and detailed in §4.1. Specifically, participants [i] provided demographic information, [ii] selected a preferred snack category, [iii] assigned long-run choice percentages to all products in that category's assortment and [iv] identified their three (ordered) favorite products from that assortment, then [v] provided information about their usage rate, attitudes, and perceptions about products in the category. The second (i.e., consumption) phase of the longitudinal experiments required participants to sequentially consume a set of five snacks. Each student participant was assigned a box with five snacks to consume one-into the classroom in a cart before each class meeting. At the beginning of class, participants chose one snack from their box for personal consumption that day. They were instructed not to trade snacks or to select a snack for someone else to consume. At the end of class, researchers removed the cart and recorded which product alternative each participant had chosen to consume. Each consumption choice reduced the participant's inventory of either their favorite or second-favorite alternative. This procedure was repeated until all snacks were consumed.

## 5.2 Longitudinal Experiment 1

Participants' set of five snacks included either: [i] four units of their favorite product alternative and one unit of their second-favorite, or [ii] one unit of their favorite and four units of their second favorite. Sixty-nine graduate students completed the first phase questionnaire; 67 undertook the second phase consumption task.

Figure 1: Experiment 1 - Percent Favorite Chosen vs. Favorite Inventory

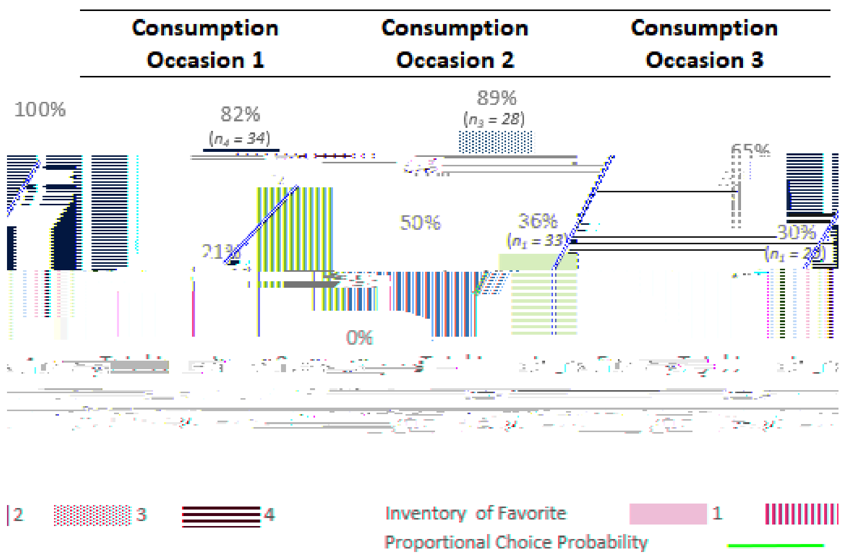
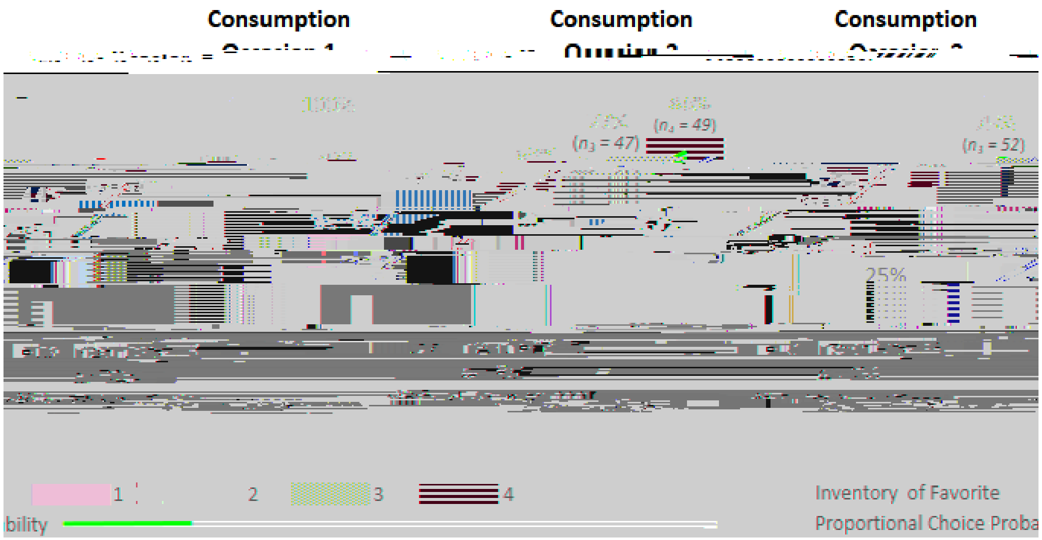



Figure 1 shows, from left to right, the sample proportion of participants who chose their favorite on the first, second, and third consumption occasions. We did not assess the last two consumption occasions because they offered no information about the relationship between inventory and consumption. The red line shows the theoretical (i.e., proportional) probability of choosing the favorite, based on the inventory of the two product alternatives available. Note that, if consumers are myopic, we would not expect the bar heights to be significantly different because consumption of the favorite would be driven by preference, not inventory. A visual inspection clearly shows that the data are not consistent with myopic behavior. On the first consumption occasion, all 67 participants chose from a set that included either one or four units of their favorite product alternative with the remainder being their second-favorite. For this consumption occasion, we find strong evidence of a relationship between inventory and consumption choice of the favorite ($\chi^2(1) = 14.72; p = 0.0001$). On the second consumption occasion, the 48 participants who still had a choice (and completed the task) chose from a set that included either one or three units of their favorite with the remainder being their second-favorite. For this consumption occasion, we find again strong evidence of a

this manipulation was not included in the two longitudinal experiments.

Figure 2: Experiment 2 - Percent Favorite Chosen vs. Favorite Inventory



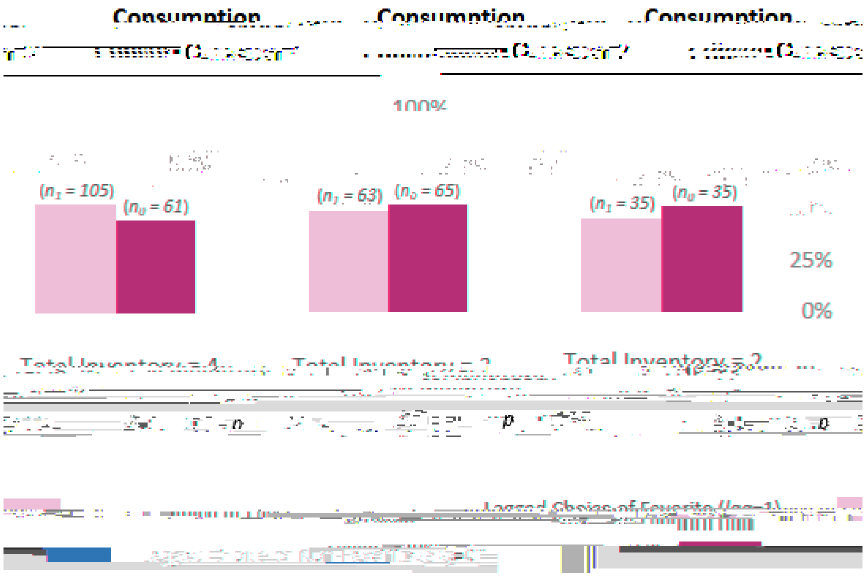

of their favorite with the remainder being their second-favorite. For this consumption occasion, we do not find sufficient evidence to confirm a relationship between inventory and consumption choice $\chi^2(1) = 2.55, p = 0.1102$, although the pattern of choices is consistent with Proposition 2's inventory-based consumption policy. It is important to note that participants were more likely to choose their favorite when they had more inventory of that favorite for every inventory configuration on every consumption occasion — additional strong support for Proposition 2 and *RMPC* theory's inventory-based consumption policy.

The inventory-based policy that we observe in the data may be intuitive, but is by no means the only potentially intuitive consumption policy. As noted earlier, myopic consumers might consume their favorite in proportion to its long-run choice probability. Alternatively, indifferent consumers might consume available product alternatives with equal probability. We observed neither of these patterns; rather we observed a consumption policy consistent with forward-looking consumers maximizing the future value of the set.

In Experiment 2, we extended the manipulation of beginning inventory levels to test for variety seeking in consumption choices. Figure 4 shows consumption choices for the second, third, and fourth consumption occasions, together with lagged choices (i.e., the previous consumption occasion's choice). Satiation, and so variety seeking, would imply a negative relationship between successive consumption choices as the consumer's utility for the product and its attributes diminishes. We find no such relationship. Specifically, tests of independence for the second $\chi^2(1) = 0.85, p = 0.3558$, third $\chi^2(1) = 0.12, p = 0.7257$, and fourth $\chi^2(1) = 0.23, p = 0.6324$ consumption occasions provide scant evidence of satiation in participants' consecutive consumption choices. The lack of variety seeking evidence stands in contrast to the compelling evidence of inventory-based consumption choices.

Following Experiment 2's consumption phase, we followed up with participants to determine which factors had affected their consumption choices. Specifically, participants were asked: "Looking back on only those days when you could choose between your favorite and second-favorite snacks, which of the following factors affected your choices?" Stochastic *preference* was identified as "which snack I felt like eating the most on that day." *Inventory* was identified as "the number of each product that was available in my box on that day." *Satiation* was identified as "which snack I had eaten recently and so was 'getting tired of.'" Participants evaluated all three factors

Figure 3: Experiment 2 - Percent Favorite Chosen vs. Lagged Favorite/Non-Favorite




using a 7-point agreement scale with 1 = \Strongly Disagree" and 7 = \Strongly Agree." For the 177 participants who responded, the mean response for current *preference* was 5.71 (indicating households that purchased a total of 8,670 single-serve yogurt cups on 1,611 shopping trips during the calibration period, then purchased 2,376 cups on 443 shopping trips during the estimation period. Although this dataset is small, it is constructed purposefully to avoid intra-household heterogeneity and so provide a clean test of Proposition 3.[15]

Data from the calibration period were used for two purposes. The first was to determine panelists' long-run consumption preferences. Those preferences were determined by UPC because the variety and ambiguity of flavors (e.g., white chocolate strawberry, cherry vanilla creme, pina colada, cookies & creme, apricot mango, lemon meringue, key lime pie, mixed berry) did not allow for a parsimonious attribute decomposition. From consumption preferences, we developed household-level utilities for UPCs. The second use of calibration data was to calculate consumption rates. Panelists did not record their consumption—such data are rare—so we we estimated consumption rates, assuming that all yogurt purchases made during the calibration period were consumed. Initially, we conjectured that each day presented a consumption opportunity. Interestingly, we found that one panelist consumed 1.328 units/day (recall that each unit is a single serving), buying yogurt on 114 shopping trips during the calibration period. All other panelists consumed an average of less than one unit/day.

Data from the calibration period were used to assess the relative variety of yogurt purchases. Consistent with RMPC theory of shopping and consumption, we assumed that $n$, the number of units chosen on a given trip, is exogenous.[16] Variety was measured as the number of product alternatives $m$ in the chosen set. Clearly, $m$ depends on the set size $n$ ($m \leq n$). To control for this dependency, we took advantage of the fact that the base RMPC model (with no outside option) is actually the limiting case of the more general model (with utility of the outside option set to $-\infty$) for a given set size $n$. To evaluate the observed variety, $m$, we therefore compared it to the variety of the base RMPC model's optimal set of the same size, $m^{opt}$, which implicitly assumes the maximum consumption rate. Using (3.1) to compute set valuations, we determined $m^{opt}$ from the base RMPC model for every panelist and every set size $n$, which we then used to determine the relative variety of observed purchases. The relative variety measure for yogurt purchases is the proportional difference between observed and optimal variety, $D = \frac{m - m^{opt}}{m^{opt}}$.[17]

Proposition 3 states that the variety included in a consumer's optimal choice set is decreasing in consumption rate; however, it does not specify a functional form for that relationship. We therefore estimated nonparametric correlations—Spearman's Rank Correlation and Kendall's Tau—as well as Pearson's $R$. An important characteristic of the data is that relative variety changed across a panelist's yogurt purchases, but the panelist's consumption rate did not. We therefore include subscripts for trip $t$ and household $h$ in the remaining exposition.

lower consumption rates will include more variety than the multi-product choices of consumers with higher consumption rates. This prediction can be explained in terms of competition. In cases where a category is seldom consumed (implying an attractive outside option), more variety in multi-product choice is the best strategy to overcome that outside option and 'win' a consumption occasion. For example, if one rarely drinks wine, having two bottles of red or two bottles of white is likely inferior to having one bottle of each. The set with greater variety offers a higher probability of including an alternative that one wants when the mood strikes. We found empirical support for this proposition using yogurt purchases in multi-market panel data.

In summary, we have presented evidence that uniformly supports a rational theory of multi-product choice based on the maximization of expected future utility. This theory represents a compelling alternative to existing theories of multi-product choice that explain observed diversification

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## Appendix A: Short Proof of the Value Function and Optimal Policy

*Proof.* The proof is by induction. The standard zero-mean Gumbel $_i$ has c.d.f. $F(x) = exp( \quad exp( \quad x \qquad ))$ where is the Euler-Mascheroni constant. The value formula (3.1) is trivially true for $n = 1$. Assume the truth of (3.1) for the case ($n$-1). Let $k^T = (k_1; : : : ; k_M)$ with $\sum_{i=1}^{M} k_i = n \quad 1$. Then the truth of the result for $n$-1 implies $V(k) = ln \quad {}^{(n \quad 1)!}$

# Appendix B: Hierarchical Bayesian Analysis of Relative Variety

The combination of a time invariant household-level predictor ($Units{=}Day_h$ or $Days{=}Unit_h$) and a time varying household-level response variable ($D_{ht}$) led us to model the data using a hierarchical random coe  cients model. The  rst-level equation was speci  ed

$$D_{ht} = \ _h + \ _{ht}$$

and the hierarchical equation was speci  ed

$$_h = \ + \ f(Units{=}Day_h) + \ _h;$$

where $f(\cdot)$ is a monotonic transformation to allow for  exibility in functional form. The parameter of interest is  , which captures the relationship between the relative variety of a choice set, $D_{ht}$, and the transformed consumption rate, $f(Units{=}Day_h)$. The resulting model was estimated in a hierarchical Bayesian framework with minimally informative priors so that the posterior estimates were driven by the data. For each model, the 25000 Markov Chain Monte Carlo iterations converged quickly after a short burn-in period and autocorrelation proved acceptable (we also \thinned the line"), resulting in a stable posterior distribution for  .

Table 6: Relative Variety Model Estimates

| Model | Predictor | DIC | MAD | MSE | Mean | [illegible] |
|---|---|---|---|---|---|---|
| [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Coefficient" models, and actually has a slightly lower *DIC* than the fourth. In terms of predictive accuracy, the "Random Coefficient ( $= 0$)" model offers posterior predictions similar to the four full "Random Coefficient" models. We therefore conclude that unmodeled individual differences explain much more variation in $D_{ht}$ than consumption rate does. On the other hand, the functional form of the relationship between consumption rate and variety matters for predictive accuracy. The "Random Coefficient" model using $ln(Days/Unit_h)$ as the predictor has the lowest *DIC* and, like the "Random Coefficient" model using $Days/Unit_h$ as the predictor, offers more accurate predictions than the other models. The superior predictive accuracy of the two models using $f(Days/Unit_h)$ as the predictor is consistent with the nonparametric correlations reported above, where the proportional difference in variety was more highly correlated with $Days/Unit$ than with $Units/Day$. Taken together, these results suggest that the relationship between relative variety and usage rate be specified as a function of $Days/Unit$. A more extensive exploration of functional form is left for future research.

Returning to the preferred model, the "Random Coefficient" model using $ln(Days/Unit_h)$ as the predictor, the parameter estimate for  is positive and significant. The mean estimate is 0.092 and, based on the posterior *cdf*, $Pr( > 0) = 0.024$. For the "Random Coefficient" models using different predictors, the posterior estimate of  is always in the expected direction — positive for $f(Days/Unit_h)$ and negative for $f(Units/Day_h)$, though the posterior estimates of  are only significant for models using $f(Days/Unit_h)$ as the predictor.