Connecting Conceptual and Procedural Knowledge

*Spaghetti Trig: Use a hands-on approach to connect trigonometric exact values to the graphs of y = sin x and y = cos x. See handout for detailed instructions.

*Football Stadium Problem: In Texas, almost every high school math student experiences a football game in a stadium. This problem connects the mathematical formulas for ellipses and a real-world situation involving stadium seating. See handout for directions.

*How Can I Get There?: Students explore paths between two different homes as a way to discover and reinforce ideas related to permutations, combinations, and Pascal's triangle. See handout for activity details.

Procedural Fluency

*Exact Value Tricks: Hints to help students memorize these all-important values.

*BINGO: [illegible] ... problems [illegible] ...ded.

Collaboration/Movement

*Scavenger Hunt: Write a problem towards the bottom of a half-sheet piece of colored paper, and the answer at the top of a different half-sheet of paper. Write another problem below that answer, and continue until there are 15-20 problems. Write the answer to the last question [illegible] first page. To play, place the problems RANDOMLY around the room, in your hallway, or around the school if you can, and put the students in groups. Students need paper to work on, and should pick a problem to start at. As they complete the problem, they search for their answer and their work [illegible] page. Students follow the trail of questions until they have completed all questions.

*Trig Sticky Note Matchup: This is a fun way to start class and get the students thinking about math from the second they arrive. On small sticky notes write an angle measure in degrees [illegible] angle measure that is coterminal. Write enough pairs so that everyone in class will receive an angle [illegible] Stand at the door as students enter and hand them a sticky note (mix up the order first!). [illegible]

[redacted]

[redacted]

It is important to note to the students at this point that we are interested in finding a function in terms of the angle θ that will give us the value of the x coordinate and a function in terms of the angle θ that will give us the value of the y coordinate. From previous knowledge students should know that the functions we are looking for are sine and cosine.

and 5π/12 and their corresponding reference angles around the unit circle.

It may be a good idea to have half of your students graph the sine function and half graph the cosine function. The activity is identical with the exception that the spaghetti will be used to measure the ordered pair's distance from the vertical axis in the unit circle.

FOOTBALL STADIUM PROBLEM

(adapted from Paul Forester, *Algebra and Trigonometry: Functions and Applications*, 2nd edition, c.1998)

Ornery and Sly Construction Company needs your help. The company has a

[redacted]

[redacted]

18. Represent the number of arrangements of the letters in the string RRR using factorial notation.

$\frac{8!}{5! \cdot 3!}$

... distinct routes.

Looking back, we find that Jane can get to her final des...

... one arrangement is possible: RRR. Thus, the number of arrangements of URRR is different from, ...

... ways to arrange the letters in the string URRR are possible. These paths are shown in figure 5. They correspond to URRR (1), RURR (2), RRUR (3), RRRU (4).

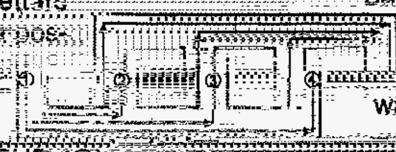


Fig. 5

... ways and to intersection L in

$\frac{7!}{4! \cdot 3!}$

Consider a different situation, shown in **figure 6**.

ways. Adding, we find that Jane can get to Bill's house in 21 + 35 = 56 distinct routes.

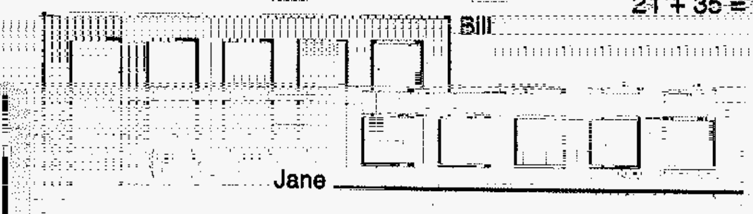


Fig. 6

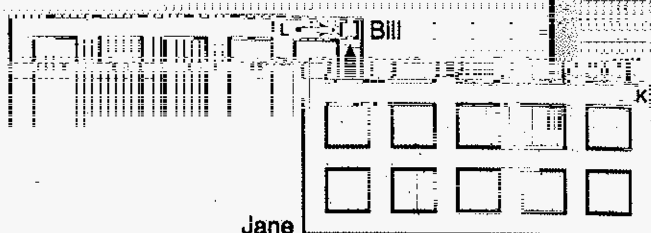


Fig. 7

19. How many blocks must Jane travel? ____ How many to the right? ____

This process is like finding the number of distinct arrangements of the string RRRRRUU.

Symbolizing this process, we find that

$\frac{8!}{5! \cdot 3!}$

20. How many arrangements of seven different letters are possible?

... can be written

... Of five different letters? Of two different letters?

$5! \cdot 2! \quad 4! \cdot 3!$

... numerators ... We follow a similar procedure for RRRRRUU. We would find 7! arrangements of the seven letters, in which 5! are repeats of the R's and 2! are repeats of the U's. As a result,

$\frac{7!}{5! \cdot 2!}$ and $\frac{7}{4! \cdot 3!}$

with the numerator of

$\frac{8!}{5! \cdot 3!}$

$\frac{7!}{5! \cdot 2!} = 21$

distinct arrangements of RRRRRUU are possible. We ...

... that Jane has twenty-one different routes to follow to get to Bill's house.

22. What do you notice in comparing the denominators of

$\frac{7!}{5! \cdot 2!} \quad \frac{7!}{4! \cdot 3!}$

We can use permutations to find the number of distinct paths ...

... blocks to travel, five of which were to the right and three were up. Using these ...

$\frac{8!}{5! \cdot 3!}$

... routes to take ...

How Can I Get There?—*Continued*

23. Use the observations in questions ... to answer the following question. If $\frac{10!}{6! \cdot 4!}$ represents the number of different routes for a complete trip, what two fractions represent the number of ways to get to the two places from which the final block can be traveled? __________ and __________

24. Verify that the sum of these two fractions is $\frac{10!}{6! \cdot 4!}$.

B and C
1 + 2 = 3
C
1

2. RUU, URU, UUR
3. RRR, 1
4. RRU, RUR, URR, 3
5. RUU, URU, UUR, 3
6. UUU, 1

Did you know that ...

• Jane's original problem is analogous to moving from the point (0, 0) to the point (6, 4) on the coordinate grid?

12.58 days

Can you ...

extend this procedure to count the ways a king can move on a chessboard? (Remember that a king can also move diagonally.)

Tricks to Remember Trigonometric Values

OIO Chart

	sinθ	cosθ	tanθ
0°	0	1	0
90°	1	0	U
180°	0	[illegible]	0

Each line reads as follows:

Oh, I owe
I owe you
Oh, I don't owe
[illegible] don't owe you

The Square Root Chart

	30°	45°	60°
sin	1	2	3
cos	3	2	1

TRIGO – Basic Trig

Choose 24 of the following 25 answers to randomly fill in your Trigo card below. Notice some of the answers repeat, but you can only cover one space at a time when you are playing!!

135° $\quad \frac{\pi}{2} \quad \frac{5\pi}{4}$

-150°

-330° $\quad$ 315°

1 $\quad$ -1 $\quad$ -1 $\quad$ 0 $\quad$ 0

undefined $\quad$ negative $\quad$ positive $\quad$ undefined

T	R	I	G	O
		FREE		

TRIGO – Basic Trig

$\tan 90°$ $\sin 180°$ $\sin \frac{\pi}{2}$ $\cos \pi$ $\sin \frac{3\pi}{2}$ $\tan 270°$

$\cos\theta$ in Quadrant II

$\tan\theta$ in Quadrant III

$\cos \frac{3\pi}{2}$ $\tan \frac{\pi}{4}$

$\cos\theta$ $\sin\theta$ $\tan\theta$ $\sec\theta$ $\csc\theta$ $\cot\theta$

change 150° to radians

change 90° to radians

change $(-120°)$ to radians

chang

change $\frac{7\pi}{}$ to degrees

change $-\frac{5\pi}{4}$ to degrees

change $\frac{3\pi}{}$ to deg

change $\frac{11\pi}{6}$ to degrees

angle coterminal to $\frac{2\pi}{3}$

angle coterminal to $-\frac{7\pi}{3}$

angle coterminal to $-\frac{\pi}{4}$

angle coterminal to $-\frac{3\pi}{4}$

angle coterminal to $\frac{11\pi}{6}$

POLO – [illegible] Polar Coordinates

Choose 24 of the following 28 answers to randomly fill in your Polo card below.

$(2, 330°)$ $(4, -150°)$ $(4, -30°)$ $(4, 240°)$ $(2, -60°)$ $(2, 225°)$

$\left(\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$ $-2i$ $-2-4i$ $\sqrt{10}$

$3-15i$ $2\sqrt{3}+2i$ $\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ $4\sqrt{2}+4\sqrt{2}i$ $\frac{1+7i}{10}$ $-7-1$ [illegible]

$\mathrm{cis}\,330°$ $2\,\mathrm{cis}\,180°$ $2\,\mathrm{cis}\,300°$ $2\,\mathrm{cis}\,135°$ $5\,\mathrm{cis}\,60°$ $30\,\mathrm{cis}\,135°$

$32\,\mathrm{cis}\,42°$ $2\,\mathrm{cis}\,(-135°)$ $8\,\mathrm{cis}\,25°$ $88\,\mathrm{cis}\,[illegible]$

P	O	☺	L	O
	FREE			

POLO

Problems:

$\sqrt{-8}\cdot\sqrt{-5}$ $(2-5i)-(-3-7i)$ $(4-7i)-(1+8i)$ $(2-4i)(-3+i)$

... $\frac{1+2i}{...}$ $\frac{8cis120°}{2cis80°}$ $\frac{35cis98°}{7cis36°}$ ... $3+5i$

$56cis57°$ ... $3+3i$ ... $7cis22°$ ... $2+4i$

... another name for $(4, -120°)$ ... change to ... $(5cis32°)(6cis41°)$

... another name for $(-3, -210°)$ ... another name for $(-4, 300°)$

...tangular form: $(-6, 135°)$ change to polar form: $(\sqrt{3}, -1)$ change to rec...

...tangular form: $(2, 60°)$ change to polar form: $(-\sqrt{2}, -\sqrt{2})$ change to rec...

...polar form: ... change to po...

...polar form: $-\sqrt{2}-\sqrt{2}i$ change to polar form: $\sqrt{3}-i$ change to ...

change to complex form: $3cis45°$ ... change to polar form: $3i$ ...

...ex form: $4cis30°$ change to complex form: $3cis(-150°)$ change to comple...

L	O	☺	G	O
		FREE		

…rs and fill in your LOGO card at random. Mark out each answer as you go so you know which answers have been used. Choose 24 of the following 28 answers.

2	6	4	−27	$\frac{2}{13}$	$\frac{1}{3}$
−4	$\frac{3}{5}$	$\frac{2}{3}$	3	$\frac{17}{3}$	$\frac{1}{32}$
−3	$\frac{11}{5}$	0	−1	$\frac{3}{2}$	$\frac{2}{7}$
15	−2	1	$\frac{5}{6}$	$\frac{1}{2}$	−5
		$\frac{1}{36}$	$\frac{1}{8}$	$-\frac{1}{4}$	64

LOGO

Problems:

$\log_3 81$ $\log_5 125$ $\log_{15} \sqrt{15^3}$ $\log_2 \frac{1}{32}$

6 $\log \frac{1}{100}$ $\log_1 16$ $\log \sqrt[3]{10}$ $\log_2 4 \cdot \log_2 1$

$\log_2 8)$ $\log_5 1 - \log_5 125$ $\left(\frac{125}{27}\right)^{-\frac{1}{3}}$ $\log_9(\log_3 27)$ $\log_3($

$32^{-\frac{3}{5}}$ $16^{-\frac{5}{4}}$ $\left(\frac{9}{\ }\right)^{-\frac{1}{2}}$ $9^{\frac{3}{2}}$

$4^{x-2} = 16^{7x}$ $\log_8 x = 2$ $\log_6 x = -2$ $\log_x \frac{36}{25} = -2$

$\log\ 27 = 3x + 6$ $25^{2x} =$ $9^{3x} =$ $\log\ 16 = 2x - 2$

$-1) + \log\ 5 = 2$ 49 $\log\ \frac{1}{\ } = x - 2$ $\log\ (x - 1) = 2$ $\log\ ($

$\log_2 (x - 3) = \log\ 5$

Around the World Sample Problem Sets

Trig Sum/Difference Identities

1) Find the exact value of $\cos(-15°)$.

2) Find $\sin(\alpha-\beta)$ if $\cos\alpha = -\frac{8}{17}$ for $\frac{\pi}{2} < \alpha < \pi$, and $\sin\beta = -\frac{7}{25}$ for $\pi < \beta < \frac{3\pi}{2}$.

3) Find the exact value of $\tan 75°$.

4) Find $\cos(\alpha-\beta)$ if $\sin\alpha = \frac{4}{5}$ for $0 < \alpha < \frac{\pi}{2}$, and $\cos\beta = -\frac{24}{25}$ for $\frac{\pi}{2} < \beta < \pi$.

5) Prove $\sin(\alpha+\beta)$ [illegible]

6) Find the exact value of $\sin 105°$.

7) Prove $\tan(\alpha+180°) = \tan\alpha$.

8) Find $\tan(\alpha-\beta)$ if $\sin\alpha = -\frac{4}{5}$ for $\pi < \alpha < \frac{3\pi}{2}$, and $\sin\beta = -\frac{7}{25}$ for $\frac{3\pi}{2} < \beta < 2\pi$.

9) Prove $\tan\left(\frac{\pi}{4}-\alpha\right) = \frac{1-\tan\alpha}{1+\tan\alpha}$

10) Prove $\cos\left(\frac{3\pi}{2}-x\right) = -\sin x$

Domain/Range/Functions

1) Graph $y = |x-1|$ for $-2 \le x \le 3$.

2) What is the domain for $f(x) = \frac{3}{x^2+x-6}$?

3) If $f(x) = x^2-1$ and $g(x) = x+1$, find $f(g(x))$.

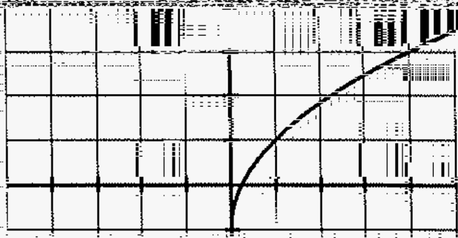
4) Find the domain and range of $f(x) = \sqrt{x}+3$.

5) What is the domain and range of $g(x) = |x+4|$.

6) What is the range of $y = x^2+5$?

7) Name the domain and range of the following graph:

8) Name the domain and range of the following graph:



9) If $f(x) = 2x^2 - x$ and $g(x) = 5x + 1$, find $(f - g)(x)$.

10) What is the domain of $y = \sqrt{x - 3}$?

$\sqrt{2}$

$\frac{\sqrt{2}}{2}$

$\sqrt{3}$

$sec60°$

$tan60°$

$-\frac{\sqrt{3}}{3}$

$csc270°$

$sec30°$

$sin45°$

$\frac{2\sqrt{3}}{3}$

$cot45°$

$\frac{1}{2}$

0

$cos30°$

1

-2

$-\frac{1}{2}$

$-\frac{\sqrt{3}}{3}$

$cot90°$

$\sqrt{3}$

$\frac{\sqrt{3}}{2}$

2

2

$tan30°$

[redacted]

[redacted]

Precalculus preAP
Trig Graph Project

name: ______________________________
date: ______________________________

You should now have had a chance to play around on the graphing calculator with the 2 student samples I showed you.

Project: 15 points ... (due 12/7/12)

You will create a picture using each of the six trigonometric functions ONLY once each.

1. Use $y = A\sin B(x - C) + D$, $y = A\cos B(x - C) + D$, $y = A\tan B(x - C) + D$ ... $y = A\csc B(x - C) + D$ ...

Create your graph on www.desmos.com (launch the calculator) ...

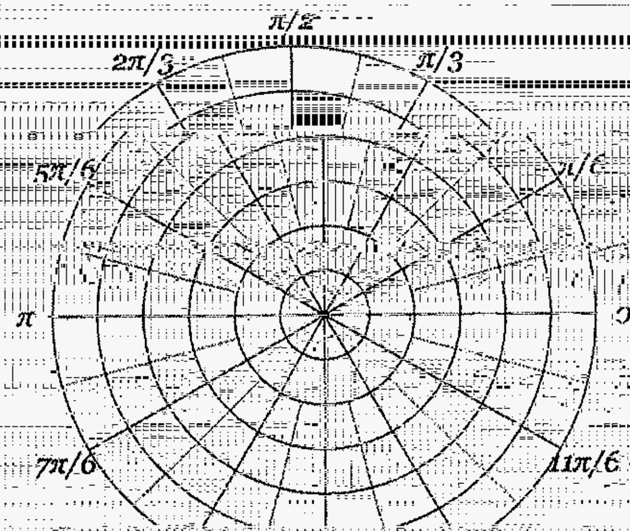
Take a screen shot of your final graph and equations ...

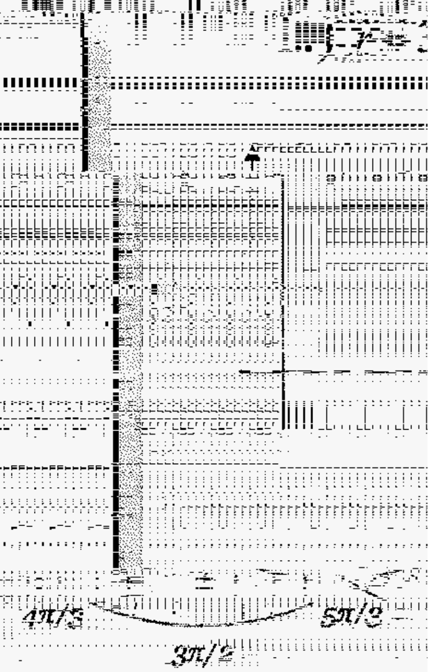
Import your picture into paint and ... You may add a FEW details to enhance the picture, but the drawing should be mostly the trig (2 points)

You will email your final picture ...

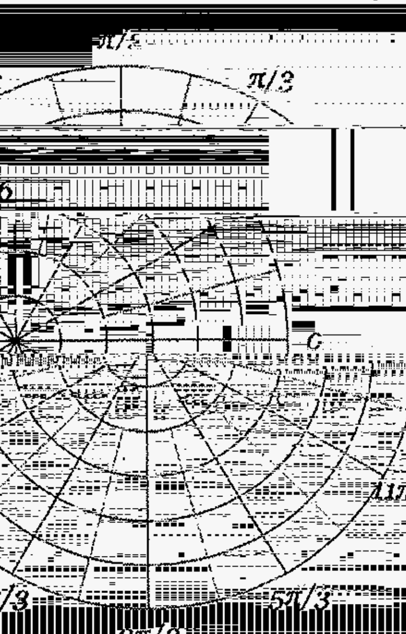
Practice with Polar Graphing

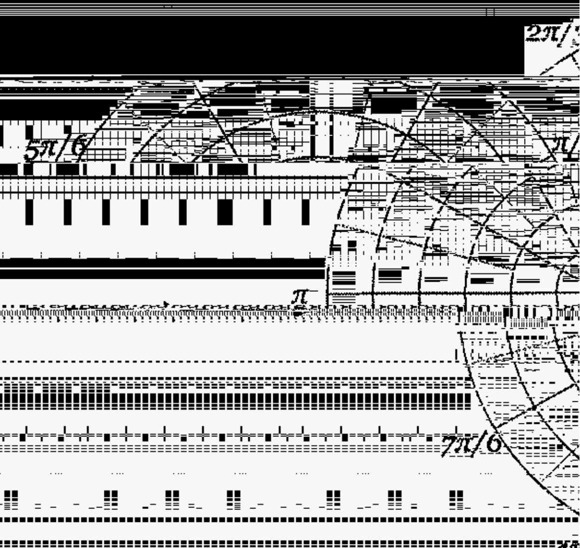
$+ 3\cos\theta$



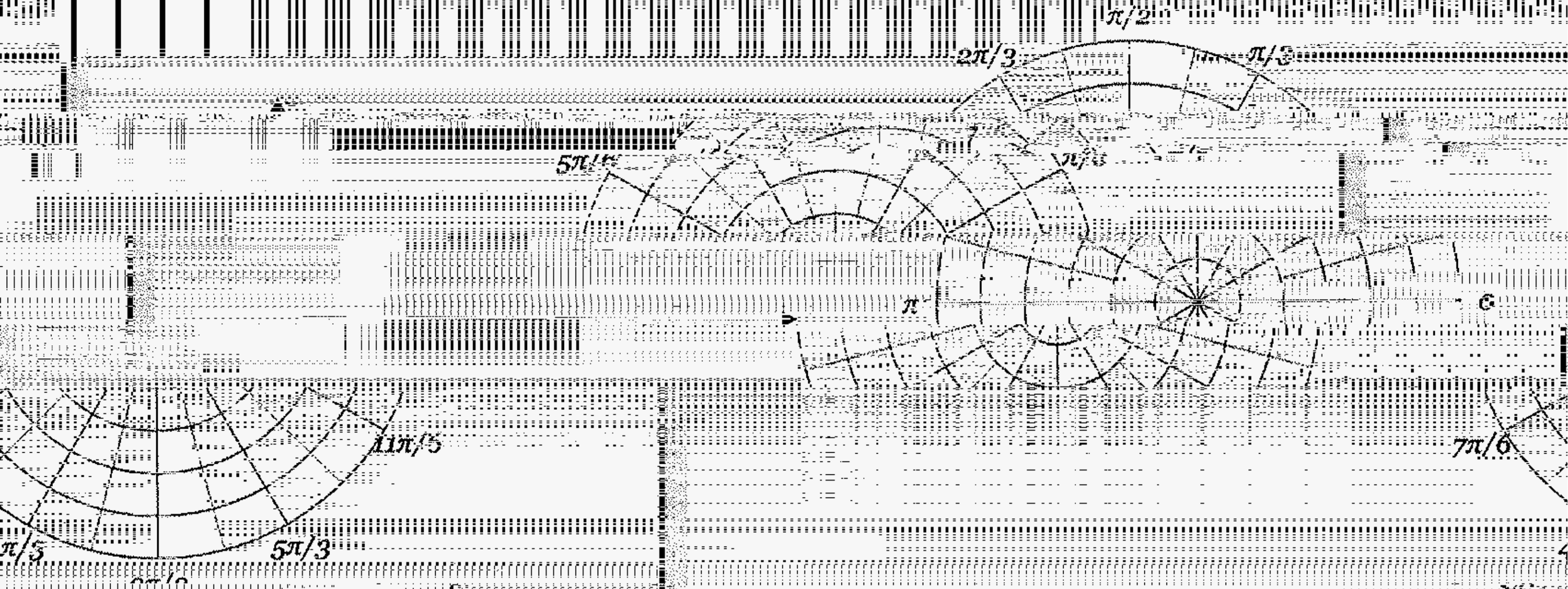


2.) $r = 1 + 3\sin\theta$





3.) $r = 2 - 3\cos\theta$



$- 4\sin\theta$

Fun with Polars Project

Use what you have learned about polar equations and anything you have derived from your own calculator explorations to produce a POLAR PICTURE.

You must use at least 6 equations. If you choose to use more than 6 equations, you may store up to 6 of them by doing the following:

1) Graph equations
2) DRAW
3) STO
4) StorePic
5) 1
6) ENTER
7) Graph remaining equations
8) DRAW
9) STO
10) Recall Pic
11) 1
12) ENTER

Hand in a neatly written page including:

MODE
WINDOW SETTINGS
STYLE OF GRAPH (connected, dotted, bold, etc)
EQUATIONS (including any stored equations)
ROUGH SKETCH OF PICTURE

Sample graphs from students:

Coding/Encryption Name___________________________

Objective: To use functions and inverse functions to demonstrate how written information (text) can be encrypted and decrypted.

Here is a CODING table for the alphabet. To code a message you would simply replace the letters of the message with the numbers from the table. Computers use a system like this to capture text data.

Encryption changes the coding of the letters according to a particular rule, hopefully known only to the person intended to receive the message.

A	1
B	2
F	6
G	7
H	8
I	9
	12
M	13
N	14
	15
	16
Q	17
R	18
S	19
	20
U	21
W	23
X	24
Y	25
	26
space	
	28
	29

from the table.

Choose a simple algebraic function, a linear function.

ENCRYPT the letters of your name according to the function.

To DECRYPT your name, you will need to use the INVERSE of your ENCRYPTING FUNCTION. Find the INVERSE FUNCTION and write it here.

See if you can successfully decrypt your name using the inverse function.

In telecommunications systems many messages are encrypted for security and privacy. Messages of financial transactions and personal data sent over the internet are encrypted, as are military communications, using functions, although the encryption functions are much more complex.

Think of a message you would like to send to a friend. I recommend no more than six words. Then write a simple function to use to encrypt it. Write them both in the space below.

In the next step, write the encrypted message along with the function. Trade sheets with a friend to decode each other's messages.

Message Sender: ____

Encrypted Message:

A	1
B	2
C	3
D	4
E	5
F	6
G	7
H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

Message Receiver: ____

Decrypted Message:

ACTIVITY #1

Here is a secret message from me. This time see if you can "crack the code" without my giving you the encoding function.

51 103 75 47 91 119 35 127 47 91

[illegible]

39 91 95 131

[illegible]	[illegible]
F	5
F	6
G	7
H	8
I	9
J	10
K	1
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29
@	30

ACTIVITY #?

Here is a secret message from me. This time see if you can "crack the code" without my giving you the encoding function.

... function is a linear function. Try looking at the differences between coded ...

H	8
I	9
J	10
K	11
L	12
M	13
N	14
O	15
P	16
Q	17
R	18
S	19
T	20
U	21
V	22
W	23
X	24
Y	25
Z	26
Space	27
!	28
?	29

Answers to Coding/Encryption Activities

Activity #1:

The equation is $y = 4x + 19$. The message is: HUNGRY? GRAB A SNICKERS!

Activity #2:

The equation is $y = 5x + 14$. The message is: YO QUIERO TACO BELL

A Leaky Bottle Experiment

- Each group needs a timekeeper, water-level reader/bottle holder, and recorder
- Fill the bottle so the water level is below the curve at the top
- When the timekeeper says "go," uncover the hole and let the water run freely
- The timekeeper calls out time every 10 seconds
- The water-level reader reads aloud the water level to the nearest millimeter
- The recorder records the data
- Stop measuring when the water level reaches about a centimeter from the hole

1) Enter the data into your calculator and look at the scatterplot. Describe in words what your graph looks like and draw a quick sketch.

2) Write a conjecture about what types of functions might fit the data.

3) Find regression equations for several types of functions that fit the data reasonably well.

4) Decide which of your models is the best fit and explain why.

5) Use your best model to predict when the container would be empty.

HAND IN ONE REPORT WITH ALL THE MEMBERS OF YOUR GROUP WRITTEN ON THE TOP.

Be sure you have explained everything thoroughly — this will be a group grade, so be sure all members are satisfied with the finished product.

Time (seconds)	Height of water
50	[illegible]
60	[illegible]
70	[illegible]
80	[illegible]
90	[illegible]
100	[illegible]
110	[illegible]
210	[illegible]
220	[illegible]
230	[illegible]
240	[illegible]
250	[illegible]
260	[illegible]
270	[illegible]

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- Research in Mathematics Education, Southern Methodist University

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